

Archimedes, A Gold Thief and Buoyancy

by
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Once upon a time (the 3rd century B.C.) there was a very wealthy king. Like most kings, Hiero of Syracuse (on the island of modern day Sicily) wore a crown as a symbol of his authority. Over the years, Hiero was made aware that his Royal Goldsmith (who made his crown from Hiero's treasury) was living a lifestyle that was beyond his means. Hiero suspected that the Royal Goldsmith was using royal gold, intended for the royal crown, to augment his personal wealth. The goldsmith was rumored to be preparing the crowns with a cheaper alloy (using a silver-gold mix) than pure gold. No one using 3rd century B.C. technology knew how to prove or disprove the speculation that the Royal Goldsmith was "stealing from the crown."

The problem of determining the gold content of the royal crown was given to Archimedes, a noted Greek mathematician and natural philosopher. Needless to say, this was not a trivial problem! Archimedes knew that silver was less dense than gold, but did not know any way of determining the relative the density (mass/volume) of an irregularly shaped crown. The weight could be determined using a balance or scale, but the only way known to determine volume, using the geometry of the day, was to beat the crown into a solid sphere or cube. Since Hiero had specified that damage to the crown would be viewed with less than enthusiasm, Archimedes did not wish to risk the king's wrath by pounding the crown into a cube and hoping that post-analysis it could be made all better again.

While in the public baths, Archimedes observed that the level of water rose in the tub when he entered the bath. He realized this was the solution to his problem and supposedly, in his excitement, he leaped up and ran naked through the streets back to his laboratory screaming "Eureka, Eureka!" (I've got it!).

Later, he demonstrated to Hiero and his court how the amount of water overflowing a tub could be used to measure a volume. His calculations indicated the goldsmith was, indeed, an embezzler. History does not record the fate of the unscrupulous artisan.

Archimedes observation has been formalized into Archimedes Principle:

"An object partially or wholly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Translation: objects more dense than water (like lead) will sink; objects less dense than water (like cork) will float; objects of the same density will remain at the same level (hover) and neither sink nor float. Objects that sink are frequently termed negatively buoyant. Objects that float are termed positively buoyant. Objects that stay stationary at depth are said to be neutrally buoyant.

Buoyancy is easiest understood by the application of "force arrow" principles or vectors. Vectors are mathematical constructs that have magnitude (like mass) and direction (towards or away from the surface). Weight is a downward force (gravity acting on mass); buoyancy is an upward force. If these two forces are balanced, then so-called neutral buoyancy (object hovers) is achieved. If

they are not balanced, the object immersed will either sink (weight greater than upward buoyant force) or float (weight less than upward buoyant force).

NOTE: As with weight and mass, divers commonly are imprecise in the use of the term buoyancy. Rigorously, buoyancy is defined as ONLY an upward force directed against the force of weight. Although commonly used in the diving community, the terms "neutrally buoyant" and "negatively buoyant" are rigorously improper; the term "positively buoyant" is redundant. Buoyancy is much easier to understand if one only considers balancing an upward force (buoyancy) and a downward force (weight). In this scheme, there is no positive or negative. We will use the term "hover" to refer to the so-called "neutrally buoyant" state. Thus, an object will float, hover or sink. If weight is greater than buoyancy, the object sinks. If buoyancy is greater than weight, the object rises. If weight and buoyancy are identical, then the object hovers ("is weightless")

EXAMPLE: When a helicopter "hovers." (Remains stationary and neither rises or sinks) the helicopter has exactly balanced the downward force of weight with the upward force of lift supplied by the turning rotor. A diver "hovers" by balancing the downward force of weight with the upward force of buoyancy.

Buoyancy-type problems involve three factors: the weight of the object being submerged, the volume of the object submerged, and the density of the liquid involved in the problem. Any two of these factors can be used to determine the third. Let's study some representative numeric examples.

ENGLISH EXAMPLE: What is the buoyancy in seawater of a piece of wood that weighs 2000 pounds & measures 6 ft x 2 ft x 3 ft?

ANSWER: Determine forces involved:

- a. The weight of wood = 2000 pounds
- b. The volume of wood = 6 ft x 2 ft x 3 ft = 36 ft³
- c. The corresponding weight of an equal volume of seawater

$$36 \text{ ft}^3 \times 64 \text{ lb} / \text{ft}^3 = 2304 \text{ lb}$$

At this point, we know that the wood object weighs less than the corresponding volume of water (the volume of seawater that would be displaced if the entire object were to be submerged), thus it will float.

Applying "force arrows:"

weight of wood	= downward force =	2000 lb.	↓
weight of water displaced	= upward force =	2304 lb.	↑

net force	= upward	=	304 lb. ↑

The object will float with a buoyant force of 304 pounds. In order to sink the object, the object would have to weigh more than an additional 304 pounds (without changing volume.) This is the amount of "push" you would have to exert on this log for it to sink. Although the object is buoyant

(i.e., there is a net force of 304 pounds pushing up on this log), it will not be completely out of the water. The density of the log can then be used to determine how much of the log will be submerged.

$$\text{Density of log} = \frac{2000 \text{ pounds}}{36 \text{ ft}^3} = 55.6 \text{ pounds/ft}^3$$

Since this log is less dense than seawater, it will float. The amount of the volume that is submerged will be determined by the ratio of the density of the log and the density of the seawater. In general:

$$\text{Ratio: Volume Submerged} = \text{Density Object} / \text{Density Liquid}$$

Substituting the value of this log & seawater:

$$\text{Ratio} = \frac{55.6 \text{ pounds / ft}^3}{64 \text{ pounds / ft}^3} = 0.869$$

So, about 87% of the log's volume will be submerged.

ENGLISH EXAMPLE: A fully suited diver weighs 200 pounds. This diver displaces a volume of 3.0 cubic feet of seawater. Will the diver float or sink?

ANSWER: Determine forces involved:

a. weight of equal volume of seawater:

$$3.0 \text{ ft}^3 \times 64 \text{ lb / ft}^3 = 192 \text{ lb.}$$

b. applying force arrows:

$$\begin{array}{rcl} \text{weight of diver} & = & 200 \text{ lb} \downarrow \\ \text{displaced weight} & = & 192 \text{ lb} \downarrow \\ \hline \text{net force} & = & 8 \text{ lb} \downarrow \end{array}$$

The diver will sink. This diver weighs 8 pounds in the water and is severely over-weighted. Removal of eight pounds will allow the diver to hover (which means the diver will have to do less work while diving; see the trim discussion.) Since the object of recreational diving is to enjoy the environment, less work translates into more bottom time and more fun!

ENGLISH EXAMPLE: A fully geared diver in a wet suit weighs 210 pounds. In fresh water, this diver with a scuba cylinder containing 500 psig needs 18 pounds of lead to hover. How much lead will this diver need when diving in a wet suit in seawater?

ANSWER: Using Force Arrows:

$$\begin{array}{r} \text{weight of diver: } 210 \text{ lb. } \downarrow \\ \text{weight of lead: } 18 \text{ lb. } \downarrow \\ \hline \text{total weight acting on the water: } 228 \text{ lb. } \downarrow \end{array}$$

To hover, the volume of water displaced by the diver must exert a buoyant force upward equal the total weight of the diver plus gear (downward force). This is the buoyant force exerted by a volume of fresh water (density = 62.4 lb./cubic foot) that weighs 228 pounds.

Determine Volume of diver:

$$\text{Density} = \text{Mass} / \text{Volume}$$

Rearranging:

$$\text{Volume} = \text{Mass} / \text{Density}$$

Substituting:

$$\text{Volume} = \frac{228 \text{ lb.}}{62.4 \text{ lbs/ft}^3} = 3.65 \text{ ft}^3$$

Now that we know the volume of the diver, we can determine (with the assumption the volume of the weight belt is negligible) the buoyant force from the seawater (density 64 lb./ cubic foot) the diver would displace:

$$3.65 \text{ ft}^3 \times 64 \text{ lb} / \text{ft}^3 = 233.6 \text{ lb.}$$

Applying force arrows:

$$\begin{array}{r} \text{buoyant force of sea water: } 234 \text{ lb. } \uparrow \\ \text{weight of diver \& gear: } 210 \text{ lb. } \downarrow \\ \hline \text{net force: } 24 \text{ lb. } \uparrow \end{array}$$

So, the diver that was comfortable with eighteen pounds of lead on the weight belt in fresh water must add 6 more pounds (for a total of 24 lb.) on the weight belt to dive in seawater.

ENGLISH EXAMPLE: A fully geared diver in a wet suit weighs 210 pounds. In seawater, this diver needs 18 pounds of lead to hover. How much lead will this diver need when diving in a wet suit in fresh water?

ANSWER: Using Force Arrows:

$$\begin{array}{r} \text{weight of diver: } 210 \text{ lb. } \downarrow \\ \text{weight of lead: } 18 \text{ lb. } \downarrow \\ \hline \text{total weight acting on the water: } 228 \text{ lb. } \downarrow \end{array}$$

To hover, the volume of water displaced by the diver must exert an upward buoyant force equal the total weight of the diver plus gear (downward force). This is the upward buoyant force exerted by the displaced volume of seawater (density = 64 lb./cubic foot) that weighs 228 pounds.

Determine Volume of diver: (Divers use weight and mass as equivalent terms)

$$\text{Density} = \text{Weight} / \text{Volume}$$

Rearranging:

$$\text{Volume} = \text{Weight} / \text{Density}$$

Substituting:

$$\text{Volume} = \frac{228 \text{ lb.}}{64 \text{ lbs/ft}^3} = 3.56 \text{ ft}^3$$

Now that we know the volume of the diver, we can determine the upward buoyant force from fresh water (density 62.4 lb./ cubic foot) the diver would displace:

$$3.56 \text{ ft}^3 \times 62.4 \text{ lb} / \text{ft}^3 = 222.1 \text{ lb.}$$

Applying force arrows:

$$\begin{array}{r} \text{buoyant force of fresh water: } 222 \text{ lb. } \uparrow \\ \text{weight of diver \& gear: } 210 \text{ lb. } \downarrow \\ \hline \text{net force: } 12 \text{ lb. } \uparrow \end{array}$$

So, the diver that was comfortable with eighteen pounds of lead on the weight belt in seawater must remove 6 pounds (for a total of 12 lb.) from the weight belt to dive in fresh water. The difference in density between fresh and seawater is the reason why different amounts of weight must be used when diving in different environments. When moving from fresh to seawater (with the same equipment configuration), divers must add weight. When moving from seawater to less dense fresh water, divers should remove weight.

METRIC EXAMPLE: A log weighing 6000 kg measures 1 m x 3 m x 2 m. Will this object sink or float in seawater (density = 1.0256 kg/l)?

ANSWER: Determine volume of object:

$$\text{Volume} = 1 \text{ m} \times 3 \text{ m} \times 2 \text{ m} = 6 \text{ m}^3$$

Convert cubic meters to liters:

$$6 \text{ m}^3 \times \frac{(100 \text{ cm})^3}{(1 \text{ m})^3} \times \frac{1 \text{ l}}{1000 \text{ cm}^3} = 6000 \text{ l}$$

Determine weight of the displaced water:

$$6,000 \text{ l} \times 1.0256 \text{ kg/l} = 6,154 \text{ kg}$$

Using force arrows:

weight of object:	6000 kg ↓
weight displaced water:	6154 kg ↑
net force:	154 kg ↑

Now we know that this object will float

METRIC EXAMPLE: How much of the log will be submerged?

ANSWER: Determine density of object:

$$\text{Density} = \text{Weight} / \text{Volume}$$

Substituting

$$\text{Density} = \frac{6000 \text{ kg}}{6000 \text{ l}} = 1.0 \text{ kg/l}$$

The amount that will be submerged is the ratio of densities:

$$\text{Ratio Submerged} = \frac{1.000 \text{ kg/l}}{1.0256 \text{ kg/l}} = 0.98$$

So, about 98% of this object will be submerged in seawater.

METRIC EXAMPLE: A wet suited diver weighs 74 kg with gear. The diver has a volume of 80 l. How much lead should the diver wear for diving in seawater (density = 1.026 kg/l)?

ANSWER: Determine weight of seawater displaced:

$$80 \text{ l} \times 1.026 \text{ kg / l} = 82.1 \text{ kg}$$

Using force arrows:

weight of diver:	74 kg	↓
weight of water displaced:	82 kg	↑
net force:	8 kg	↑

Since there is a resultant buoyant force of 8 kg, the diver will have to wear 8 kg to compensate.

Divers wearing wet or dry suits have an additional factor to consider. Within the wet suit are trapped bubbles of gas; a dry suit diver has air spaces between the diver and the suit. This gas (in fact, all air spaces) is subject to changes in volume as a result of changes in pressure (See Boyle's Law). This means that as the diver moves up or down in the water column, the volume of these gas spaces changes. This change in gas volume affects the diver's buoyancy. As a diver descends, the volume of the gas decreases. Thus, less water is displaced. The diver is less buoyant and sinks. On ascent, the pressure on the diver decreases. The gas expands and occupies a larger volume. This displaces more water and increases the buoyant (upward) force.

Archimedes principle points out that if we are not hovering, we MUST be either floating (moving up) or sinking. So, unless our buoyancy and weight are equal, we must expend energy to hover in the water column. However, if the buoyant force exactly matches the downward force contributed by the weight of the object submerged, then a "weightless" state is achieved. This is why NASA uses underwater training for its astronauts. By finely tuning the buoyancy of a space-suited astronaut underwater, the weightless environment of space can be simulated. This allows astronauts the opportunity to practice (using the philosophy that "Perfect Practice and Prior Planning Precedes Perfect Performance!") their mission on earth to insure success in space.

LIFTING

The lift associated with air spaces can be used to raise objects from the bottom. Since air weighs very little compared to the weight of the displaced water, it can be assumed that the lifting capacity is equal to the weight of the volume of water that is displaced by the air volume of the lifting device.

ENGLISH EXAMPLE: You wish to lift a 300-pound anchor from the bottom of a lake bed. The bottom is hard and flat (so no excess lift will be needed to overcome the suction associated with being immersed in the bottom muck). You have access to 55 gal drums (weighing 20 pounds each) that have been fitted with over-expansion vents. How many 55 gal drums will it take to lift the anchor?

ANSWER: Determine forces involved:

a. Determine weight of water displaced:

$$\text{Weight} = \text{Density} \times \text{Volume}$$

Lake implies fresh water: density = 62.4 lbs/ft³

$$\text{weight} = 55 \text{ gal} \times \frac{0.134 \text{ ft}^3}{\text{gal}} \times \frac{62.4 \text{ lb.}}{\text{ft}^3} = 459.9 \text{ lb.}$$

Applying force arrows:

$$\begin{array}{rcl} \text{weight of displaced water} & = & 460 \text{ lb. } \uparrow \\ \text{weight of drum} & = & 20 \text{ lb. } \downarrow \\ \hline \text{net force} & = & 440 \text{ lb } \uparrow \end{array}$$

Since the object to be lifted weighs less than the 440-pound lifting capacity of a 55 gal drum, a single 55 gallon drum should be sufficient to lift the 300 pound anchor. In practice, large lifting objects (like a 55 gal drum) have a large surface area and will generate considerable drag (which decreases lifting capacity). Without getting mathematically rigorous and calculating drag coefficients, a safe rule of thumb is to assume about 0.75 of the calculated lifting capacity for the lifting device in an actual lifting operation.

PROBLEM: Which weighs more underwater: a pound of lead or a pound of concrete?

ANSWER: Although both weigh the same on the surface, lead will weigh more while totally submerged. Rationale: Lead is more dense than concrete, thus an equal weight will displace less volume of water. Lead will therefore have less buoyancy counteracting its weight and thus its underwater weight will be greater.

TRIM

As a diver moves in the water column, the diver is subject to a number of forces. In the vertical plane, gravity (weight) tends to make the diver descend and buoyancy (from too little weight or too much air in the b.c.d.) makes the diver ascend. In the horizontal plane, the diver moves forward propelled by the force of the kick. The thrust, or forward motion, must overcome drag (or friction) that the diver and equipment present to the water. A good diver tries to adjust diving style to balance the forces involved.

Part of the unique exhilaration of diving is the ability to glide, weightless, under the surface of the water. It is the most efficient and enjoyable way to dive. If the diver is over weighted (a too common occurrence), then s/he must continually expend energy to overcome gravity and remain at constant depth. If the diver is under weighted, s/he must also continually expend energy in an attempt to overcome buoyancy with leg power. (In battles with the forces of buoyancy and weight, these forces always overcome leg power and fatigue is certain.) The way to maximize efficiency (decrease work load and thus increase enjoyment) in the water is to balance weight and buoyancy so that the thrust from the fins can be directed towards forward movement, not towards overcoming buoyancy errors.

Assuming a horizontal position in the water can reduce drag. The more horizontal the diver, the less drag (resistance to movement caused by friction between the diver and the dense water environment) will occur and the easier underwater swimming will be! In general, cutting the cross-sectional area by a factor of two requires four times less energy to go the same distance. A more horizontal position presents a smaller area to the path of movement and thus lessens resistance.

Bottom line:

Understanding the interaction of the forces of weight and buoyancy will help a diver achieve weightless diving. The concepts associated with buoyancy control are a superb example of the Easy Diver's principle of "Dive with your brains, not your back!"

About the author:

Larry "Harris" Taylor, Ph.D. is a biochemist and Diving Safety Coordinator at the University of Michigan. He has authored more than 100 scuba related articles. His personal dive library (See Alert Diver, Mar/Apr, 1997, p. 54) is considered one of the best recreational sources of information In North America.